

# Maximizing Op Amp Summing Power

by Steve M. Gehlbach, Ph.D.

October 9, 2005

An approach is presented that allows the design equations for the non-inverting input of a summing op amp to be as trivial as those for the inverting input, so that a single summing op amp can be used without complex design equations.

## A. Background

The summation of both inverting and non-inverting signals in a single op amp is useful for situations which require minimal circuitry or for which technical requirements demand minimal time skew between all signals. However, to do so normally requires complex design equations at the non-inverting input. A novel technique, first reported by R. G. Constanty in *Electronics* in 1972 [2], adds two additional resistors  $R^S$  and  $R^A$ , that allow the gain calculation at the non-inverting input to be as simple as the inverting input. In the original *Electronics* article, Constanty reported the results, but did not present the derivation of the equations for this very clever technique. In this authors experience over twenty plus years in engineering, this useful and novel approach is not well known among analog designers.

## B. Theory and Derivation

The gain calculations for summing signals at the inverting input of an op amp are trivial, since the inverting input of an idealized op amp is a virtual earth, so that the inputs do not interact. The gain at each input can be calculated independently of both other inverting inputs and any inputs on the non-inverting side. The gain at the  $k$ th inverting input,  $\mathbf{K}_{Ik} = \mathbf{E}_o / \mathbf{E}_{Ik}$ , is calculated using KCL for an idealized op amp as follows:

$$\mathbf{K}_{Ik} = \frac{R_f}{R_{Ik}} \quad (1)$$

where  $R_F$  is the feedback resistor and  $R_{Ik}$  is the input resistor for the  $k$ th inverting input. Note that  $\mathbf{K}_{Ik}$  represents the absolute value of the gain. See **Figure 1**.

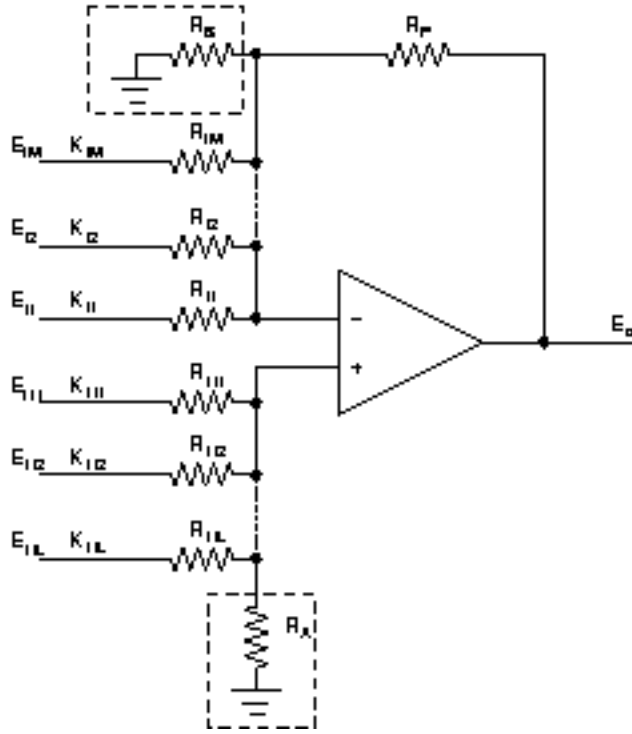


Figure 1: Summing Op Amp

At the non-inverting input, the situation is much more complex since this is the high impedance input. For the circuit in Figure 1, where there are  $M$  inverting inputs and  $L$  non-inverting inputs, using KCL for an idealized op amp, the gain  $\mathbf{K}_{Nk} = \mathbf{E}_o / \mathbf{E}_{Nk}$  at the  $k$ th non-inverting input can be calculated as (without  $R_S$  and  $R_A$ , more about that later):

$$\mathbf{K}_{Nk} = \frac{\frac{1}{\sum_{\substack{i=1 \\ i \neq k}}^L \frac{1}{R_{Ni}}}}{\frac{1}{\sum_{\substack{i=1 \\ i \neq k}}^L \frac{1}{R_{Ni}}} + R_{Nk}} \cdot \left( 1 + \frac{R_F}{\sum_{i=1}^M \frac{1}{R_{Ii}}} \right) \quad (2)$$

This can be simplified using Eq. 1:

$$K_{Nk} = \frac{1 + \sum_{i=1}^M K_{Ii}}{1 + R_{Nk} \cdot \sum_{\substack{i=1 \\ i \neq k}}^L \frac{1}{R_{Ni}}} \quad (3)$$

Rearranging and solving for  $R_{Nk}$ , we get:

$$R_{Nk} = \frac{1 + \sum_{i=1}^M K_{Ii} - K_{Nk}}{K_{Nk} \cdot \sum_{\substack{i=1 \\ i \neq k}}^L \frac{1}{R_{Ni}}} \quad (4)$$

The  $K_{Ni}$  and  $K_{Ii}$  gains are specified by the design, resulting in  $L$  equations in  $L$  unknowns in order to find each  $R_{Nk}$ . The  $R_{Ik}$ 's are calculated from Eq. (1) independently. In principal, Eq. (4) can be solved, providing a solution exists. However, it is very cumbersome for a designer to have to solve  $L$  equations in  $L$  unknowns for an otherwise simple analog circuit.

The contribution of Konstanty, is to add two additional degrees of freedom,  $R_A$  and  $R_S$ . An additional constraint is added, that the DC resistance to ground at both inputs is equal. Although we are assuming an idealized op amp in this article, setting these DC resistances equal allows minimum DC drift with changes in temperature. In modern op amps, this is not a large consideration, but it does lead to a very intriguing result.

Adding in  $R_A$  and  $R_S$ , setting the conductance to ground at the non-inverting input equal to the inverting input, we get:

$$\sum_{i=1}^L \frac{1}{R_{Ni}} + \frac{1}{R_A} = \sum_{i=1}^M \frac{1}{R_{Ii}} + \frac{1}{R_S} + \frac{1}{R_F} \quad (5)$$

If we rearrange and solve for  $R_A$ , we now have:

$$R_A = \frac{R_F}{1 + R_F \cdot \sum_{i=1}^M \frac{1}{R_{Ii}} - R_F \cdot \sum_{i=1}^L \frac{1}{R_{Ni}} + \frac{R_F}{R_S}} \quad (6)$$

Having done this, though, what is the resulting gain at the non-inverting input? Interestingly, it becomes highly simplified. Returning to Eq. (3), we add in the effects of  $R_A$  and  $R_S$ , and after some algebra:

$$K_{Nk} = \frac{1 + \sum_{i=1}^M K_{Ii} + \frac{R_F}{R_S}}{1 + R_{Nk} \cdot \sum_{\substack{i=1 \\ i \neq k}}^L \frac{1}{R_{Ni}} + \frac{R_{Nk}}{R_A}} \quad (7)$$

This can be simplified slightly to:

$$K_{Nk} = \frac{1 + \sum_{i=1}^M K_{Ii} + \frac{R_F}{R_S}}{R_{Nk} \cdot \sum_{i=1}^L \frac{1}{R_{Ni}} + \frac{R_{Nk}}{R_A}} \quad (8)$$

If we now substitute in the value for  $R_A$  from Eq. (6), the gain at the  $k$ th non-inverting input becomes a very simple formula similar to Eq (3):

$$K_{Nk} = \frac{R_F}{R_{Nk}} \quad (9)$$

The result is very useful. To restate, by adding additional degrees of freedom  $R_s$  and  $R_A$ , and constraining the input resistance to be equivalent at the inverting and non-inverting inputs, we eliminate the need for simultaneous equations to calculate the gain at the non-inverting input. As a result, calculating the gain at the non-inverting input is a simple as at the inverting input.

Returning to our constraint that led to this, Eq. (6), we can rewrite it as:

$$R_A = \frac{R_F}{1 + \sum_{i=1}^M K_{Ii} - \sum_{i=1}^L K_{Ni} + \frac{R_F}{R_S}} \quad (10)$$

In order for  $R_A$  to have a solution, the denominator must be positive or zero, in the latter case  $R_A$  would be  $\infty$ . To simplify the circuit, we can consider the case where

$$1 + \sum_{i=1}^M K_{Ii} > \sum_{i=1}^L K_{Ni} \quad (11)$$

In this event, we can set  $R_s$  to  $\infty$ , and then  $R_A$  is:

$$R_A = \frac{R_F}{1 + \sum_{i=1}^M K_{Ii} - \sum_{i=1}^L K_{Ni}} \quad (12)$$

$$\text{if } 1 + \sum_{i=1}^M K_{Ii} > \sum_{i=1}^L K_{Ni} \text{ and } R_S = \infty$$

For the case where Eq. (11) is not true, there is the possibility that  $R_A$  can be set to  $\infty$ . Setting the denominator of Eq. (10) to zero and rearranging, we get:

$$R_S = \frac{R_F}{\sum_{i=1}^L K_{Ni} - \sum_{i=1}^M K_{Ii} - 1} \quad (13)$$

$$\text{if } 1 + \sum_{i=1}^M K_{Ii} < \sum_{i=1}^L K_{Ni} ; R_A = \infty$$

In any event, Eq. (10) is always controlling, to achieve the simple result of Eq. (9). The only constrain is (quite obviously) that the denominator of Eq. (10) must be positive or zero for  $R_A$  to have a realizable value.

These results are summarized in **Figure 2**.

Component calculations for Figure 1:

$$R_{Ik} = \frac{R_F}{K_{Ik}}$$

$$R_{Nk} = \frac{R_F}{K_{Nk}}$$

providing,

$$R_A = \frac{R_F}{1 + \sum_{i=1}^M K_{Ii} - \sum_{i=1}^L K_{Ni} + \frac{R_F}{R_S}}$$

**Simplifications:**

(1) if  $1 + \sum_{i=1}^M K_{Ii} - \sum_{i=1}^L K_{Ni} > 0$

then set  $R_S = \infty$  and

$$R_A = \frac{R_F}{1 + \sum_{i=1}^M K_{Ii} - \sum_{i=1}^L K_{Ni}}$$

(2) if  $1 + \sum_{i=1}^M K_{Ii} - \sum_{i=1}^L K_{Ni} < 0$

then set  $R_A = \infty$  and

$$R_S = \frac{R_F}{\sum_{i=1}^L K_{Ni} - \sum_{i=1}^M K_{Ii} - 1}$$

(3) if  $1 + \sum_{i=1}^M K_{Ii} - \sum_{i=1}^L K_{Ni} = 0$

then  $R_A = R_S$

*Figure 2. Summarizing the equations for Figure 1 using the Konstanty technique.*

### C. Practical Example

It is common in the connection of TV and HDTV components to computer systems to convert from the HDTV standard video, YPbPr to the VGA standard, which is RGB. This conversion, called transcoding or colorspace conversion, is done by inverting the relationship matrix specified by Recommendation BT. 601 of the ITU Radiocommunications Sector (formerly CCIT). See Poynton, *Digital Video and HDTV*, p304, Eq. (25.3) [3]. The matrix relationship between these signals is:

$$\begin{bmatrix} Y \\ P_B \\ P_R \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.169 & -0.331 & 0.5 \\ 0.5 & -0.419 & -0.0813 \end{bmatrix} \cdot \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (14)$$

To transcode YPbPr into RGB, use the inverse decoding matrix [3, p305]:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.00 & 0.00 & 1.40 \\ 1.00 & -0.344 & -0.714 \\ 1.00 & 1.77 & 0.00 \end{bmatrix} \cdot \begin{bmatrix} Y \\ P_B \\ P_R \end{bmatrix} \quad (15)$$

Using the equations from Figure 2, three op amps are used to implement the color space conversion of Eq. (15), as shown in Figure 3. First the feedback resistor  $R_F$  is chosen, then the values for each input resistor are calculated, based on Eq (15). Note that a total gain of two is assigned for each amplifier, so that after the 75 ohm termination, the throughput gain is unity. For example, consider the Green amplifier, A2:

$$\begin{aligned} K_{I0} &= |-0.344 \cdot 2| = .688; & K_{I1} &= |-0.714 \cdot 2| = 1.428; & K_{N0} &= 1 \cdot 2 = 2 \\ \sum_M K_{Ii} &= 2.116; & \sum_L K_{Ni} &= 2; & 1 + \sum_M K_{Ii} - \sum_L K_{Ni} &= 1.116 > 0 \\ \text{choose } R_F &= 887; & \text{from Eq. (11)} & R_S = \infty \\ R_{N1} &= \frac{887}{2} = 443.5 \\ R_{I1} &= \frac{887}{.688} = 1289.2 \\ R_{I2} &= \frac{887}{1.428} = 621.1 \\ R_A &= \frac{887}{1.116} = 794.8 \end{aligned} \quad (16)$$

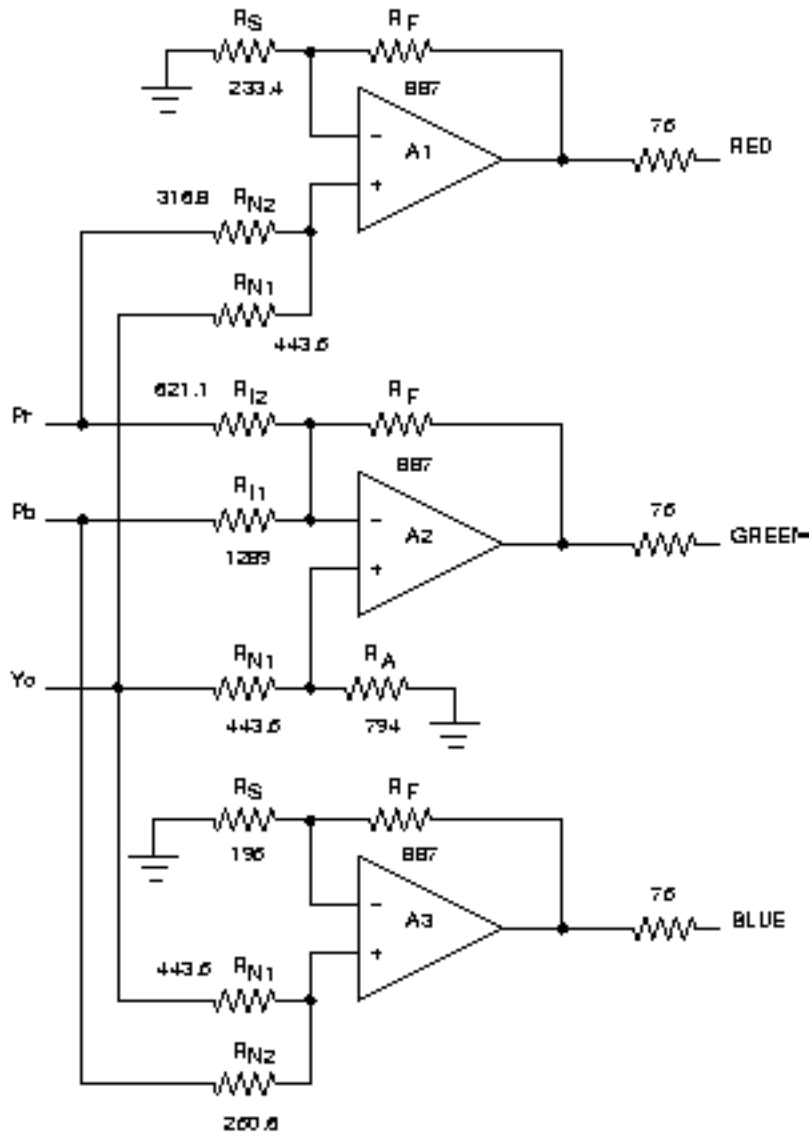


Figure 3: Colorspace conversion from YPbPr to RGB

The circuit of Figure 3 is somewhat idealized, since the inputs YPbPr are assumed to be from zero ohm sources, and there is no input termination. Furthermore, issues of DC levels and sync signals are ignored.

#### D. Transcoder for the Xbox

In 2001 Microsoft introduced a game console called the Xbox that was in fact a very advanced x86 compatible embedded computer [1]. It was designed to drive a home television, and foresaw the coming of HDTV by providing an adapter to drive YPbPr inputs of standard HDTV. However, many in the hacking community found ways to use the Xbox as a general purpose computer, and wanted an output to drive

conventional VGA displays. This author developed the first Xbox YPbPr to VGA converter and published it on the xboxhacker.net website in January of 2002. The schematic is shown in Figure 4, and the parts list is shown in Table 1.

This circuit follows the same design as in Figure 3, with the following exceptions. The output of the Xbox is designed to drive 75 ohm terminations. Therefore, the circuit in Figure 4 terminates the inputs in 75 ohms, and the calculations subtract 37.5 ohms (the Thevenin equivalent of the terminated source) from each input resistor. The amplifiers then assume a gain of two on top of the gains specified by the transcoding matrix of Eq. (15), as was done in Figure 3, to compensate for termination in 75 ohms.

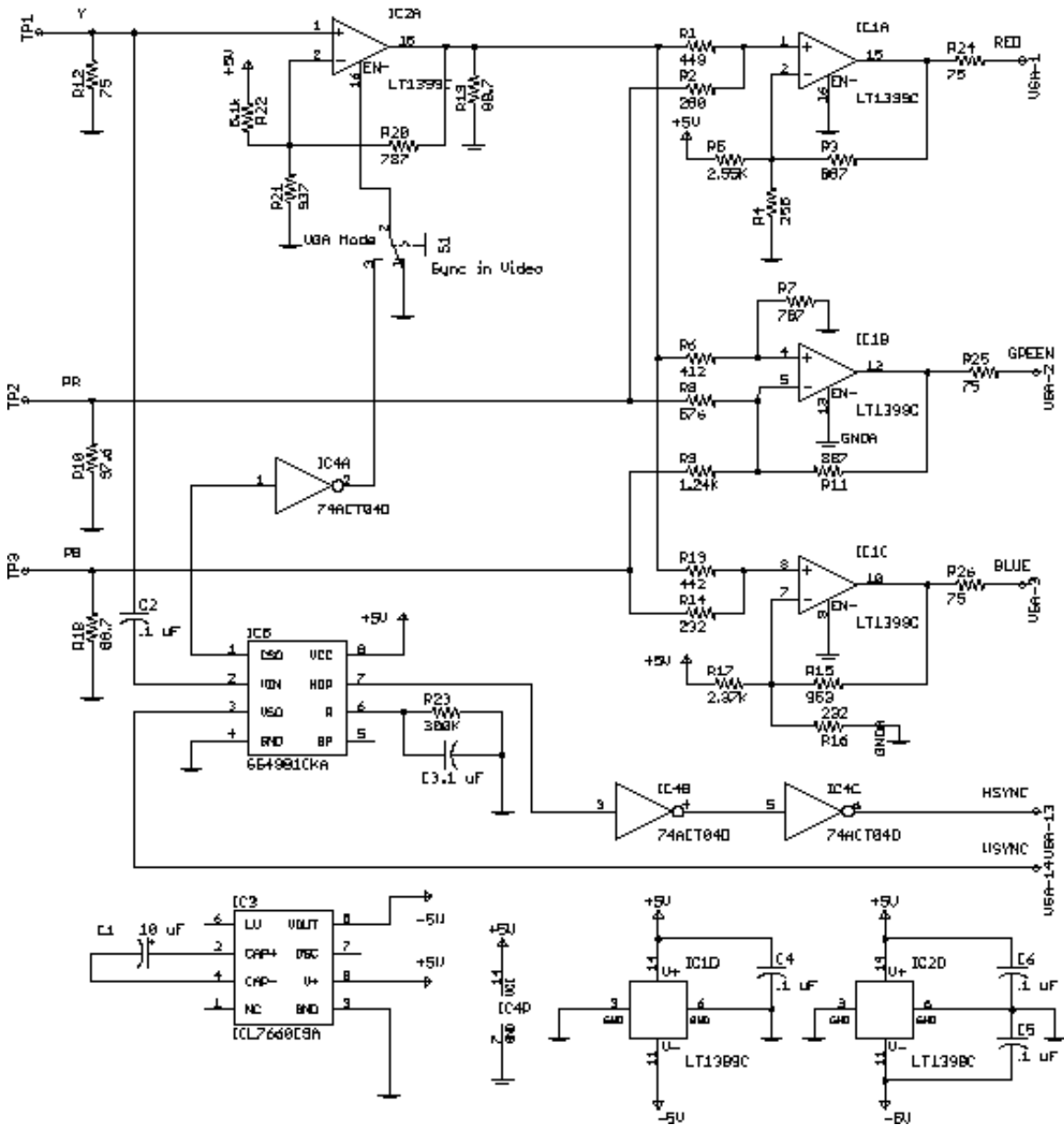


Figure 4: Xbox VGA Transcoder Schematic

<b>Part</b>	<b>Value</b>	<b>Device</b>	<b>Package</b>	<b>Description</b>
C1	10 uF	CPOL- US153CLV- 1010	153CLV- 1010	POLARIZED CAPACITOR
C2	.1 uF	C-USC1206	C1206	CAPACITOR
C3	.1 uF	C-USC1206	C1206	CAPACITOR
C4	.1 uF	C-USC1206	C1206	CAPACITOR
C5	.1 uF	C-USC1206	C1206	CAPACITOR
C6	.1 uF	C-USC1206	C1206	CAPACITOR
IC1	LT1399C	LT1399CGNGN	SSOP16	TRIPLE OP AMP
IC2	LT1399C	LT1399CGNGN	SSOP16	TRIPLE OP AMP
IC3	ICL7660CSA	ICL7660CSA	SO08	VOLTAGE CONVERTER
IC4	74ACT04D	74ACT04D	SO14	Hex INVERTER
IC5	GS4981CKA	GS4981CKA	SO8	SYNC SEPARATOR
R1	449	R-US_M0805	M0805	RESISTOR
R2	280	R-US_M0805	M0805	RESISTOR
R3	887	R-US_M0805	M0805	RESISTOR
R4	255	R-US_M0805	M0805	RESISTOR
R5	2.55K	R-US_M0805	M0805	RESISTOR
R6	412	R-US_M0805	M0805	RESISTOR
R7	787	R-US_M0805	M0805	RESISTOR
R8	576	R-US_M0805	M0805	RESISTOR
R9	1.24K	R-US_M0805	M0805	RESISTOR
R10	97.6	R-US_M0805	M0805	RESISTOR
R11	887	R-US_M0805	M0805	RESISTOR
R12	75	R-US_M0805	M0805	RESISTOR
R13	442	R-US_M0805	M0805	RESISTOR
R14	232	R-US_M0805	M0805	RESISTOR
R15	953	R-US_M0805	M0805	RESISTOR
R16	232	R-US_M0805	M0805	RESISTOR
R17	2.37K	R-US_M0805	M0805	RESISTOR
R18	88.7	R-US_M0805	M0805	RESISTOR
R19	88.7	R-US_M0805	M0805	RESISTOR
R20	787	R-US_M0805	M0805	RESISTOR
R21	937	R-US_M0805	M0805	RESISTOR

<b>Part</b>	<b>Value</b>	<b>Device</b>	<b>Package</b>	<b>Description</b>
R22	5.1k	R-US_M0805	M0805	RESISTOR
R23	300K	R-US_M0805	M0805	RESISTOR
R24	75	R-US_M0805	M0805	RESISTOR
R25	75	R-US_M0805	M0805	RESISTOR
R26	75	R-US_M0805	M0805	RESISTOR
S1		M251	M251	SLIDING SWITCH
TP1	TPTP20R	TPTP20R	TP20R	Test Point
TP2	TPTP20R	TPTP20R	TP20R	Test Point
TP3	TPTP20R	TPTP20R	TP20R	Test Point
VGA-1	TPTP20R	TPTP20R	TP20R	Test Point
VGA-2	TPTP20R	TPTP20R	TP20R	Test Point
VGA-3	TPTP20R	TPTP20R	TP20R	Test Point
VGA-13	TPTP20R	TPTP20R	TP20R	Test Point
VGA-14	TPTP20R	TPTP20R	TP20R	Test Point

*Table 1: Parts List for Schematic of Figure 4*

A few comments are appropriate regarding the design of Figure 4. The sync separator used is the Genum GS4981 instead of the more commonly available LM1881. The reason for this is the horizontal output. The LM1881 does not produce true horizontal drive, i.e., a horizontal output without vertical sync included. The VGA standard requires this, and some VGA monitors will not sync or set the reference black level properly without true horizontal drive.

Further along these lines, the VGA standard does not allow sync in the video, and the Xbox Y output contains sync. Since this signal is contained by addition in the R, G, and B, all of these will contain sync unless it is deleted prior to IC1A,B and C. This is accomplished using the gating function of the LT1399 and IC4A. A switch S1 is employed to allow the user to select whether sync in the video is desired. R19 is a discharge resistor that brings the output of IC2A quickly to ground when it is gated off to block the sync contained in the Y signal.

Another peculiarity of the Xbox YPbPr signals is that they contain a small amount of DC, about 365 millivolts. R5 and R17 compensate for this, in IC1A and IC1C, but the net offset essentially cancels in the Green signal and so no DC shift is required in IC1B.

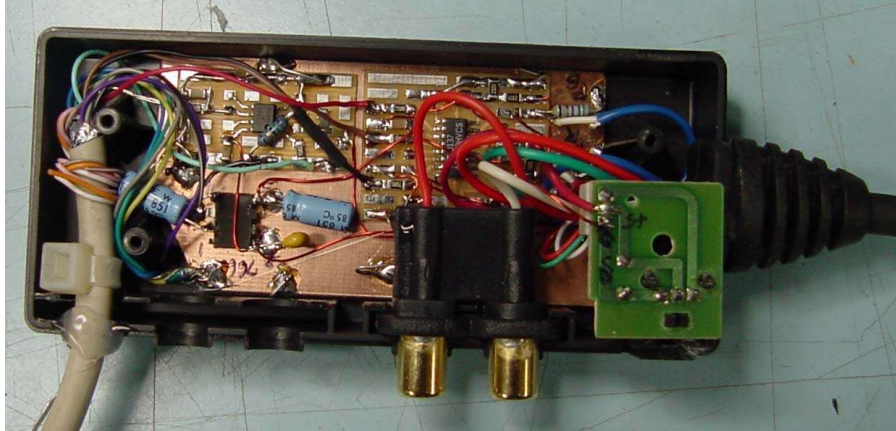


Figure 5: Prototype of Xbox VGA Transcoder

A prototype of the original Xbox transcoder constructed and tested in January 2002 is shown in Figure 5.

## E. References

1. Huang, Andrew (2003). *Hacking the Xbox*. San Diego: Xenatera Press.
2. Konstanty, Raymond G (1972). Doubling Op Amp Summing Power. *Electronics Magazine*, February 14, 1972, pp 73-75.
3. Poynton, Charles (2003). *Digital Video and HDTV*. San Francisco: Morgan Kaufmann Publishers.

## F. Document Methodology

This document was produced entirely on Linux, using a Debian derived release, installed from the Knoppix CD to a hard drive. The document was composed using OpenOffice.org v1.1.3. The equations were composed in line using the OpenOffice equation composer. Figures 1 and 3 were created using the Linux tool `xfig`, and exported to `.eps` format for inclusion in the document as a graphics object.

The schematic was created on Linux using the EAGLE Layout Editor from Cadsoft Computer (<http://www.cadsoft.de/>). EAGLE allows user level programs, which greatly assist in exporting schematics and PC layouts to other formats. For Figure 4, the schematic was exported to `.eps` using a slightly modified version of `epsdraw_411.ulp` by Filip Kinovic ([ftp://ftp.cadsoft.de/pub/userfiles/ulp/epsdraw\\_411.ulp](ftp://ftp.cadsoft.de/pub/userfiles/ulp/epsdraw_411.ulp)). This ULP (User Level Program) was modified to change the scale factor from `.5` to `1.0`. From the `eps` file, which has no preview, it was converted to `xfig` format using `ps2edit`, and then to `eps` with preview and `pdf` using `fig2dev`. A bash file to do this was created as follows:

```

#
# convert a file exported to eps from eagle using epscdraw_411m.ulp
# (which has no preview)
# --> to .fig, for use in xfig
# --> to _p.eps to add a monochrome tiff preview
# --> to .pdf (acrobat format)
# --> to .jpg jpeg format
#
if [[ $* < 1 ]]; then
    echo "usage: $0 fname.eps"
    echo "  creates fname.fig, and fname_p.eps with preview, fname.pdf,
fname.jpg"
    exit 1
fi
file=$1
#
# convert to an xfig file
#
echo convert to an xfig file
echo " ->pstoedit -f fig $file ${file}/%.eps/.fig)"
pstoedit -f fig $file ${file}/%.eps/.fig}

#
# convert xfig file to an epsi (eps with binary preview)
#
echo "convert xfig file to an epsi (eps with binary preview)"
echo " ->fig2dev -L eps -T ${file}/%.eps/.fig} ${file}/%.eps/_p.eps}"
fig2dev -L eps -T ${file}/%.eps/.fig} ${file}/%.eps/_p.eps}

#
# make a pdf file
#
echo make a pdf file
echo "fig2dev -L pdf -b 50 -z A ${file}/%.eps/.fig} ${file}/%.eps/.pdf}"
fig2dev -L pdf -b 50 -z A ${file}/%.eps/.fig} ${file}/%.eps/.pdf}

#
# make a jpeg file
#
echo make a jpeg file
echo "fig2dev -L jpeg -z A ${file}/%.eps/.fig} ${file}/%.eps/.jpg}"
fig2dev -L jpeg ${file}/%.eps/.fig} ${file}/%.eps/.jpg}

```

The `html` file was created using the OpenOffice.org “Save As” feature. However, the `eps` files are incorporated directly with this feature, and are not viewable on most Windows machines. So the `eps` files were converted to `jpeg` using the above shell script and the `html` file was hand edited to make the change.

The bill of materials was created from Eagle using a bom ULP called `bom3_csv_v2.ulp` by Benjamin West and others ([ftp://ftp.cadsoft.de/pub/userfiles/ulp/bom3\\_csv\\_v2.ulp](ftp://ftp.cadsoft.de/pub/userfiles/ulp/bom3_csv_v2.ulp)), which exports the bom as a comma separated values file, which was then imported as a table in OpenOffice.

The Acrobat (pdf) file of this document was created using the OpenOffice "Export as PDF" menu item.